(13) Evaluate $\underset{x \to 0}{Lt} \dfrac{x\cos x - \log(1+x)}{x^2}$

Ans. → $\underset{x \to 0}{Lt} \dfrac{x\cos x - \log(1+x)}{x^2} \left[\dfrac{0}{0}\right]$

Hence from L-Hospital's Rule.

$$= \lim_{x \to 0} \frac{x \times -\sin x + \cos x \times 1 - \frac{1}{(1+x)} \times 0}{2x}$$

$$= \lim_{x \to 0} \frac{-x\sin x + \cos x - \frac{1}{1-x}}{2x} \left[\frac{0}{0}\right]$$

Hence from L-Hospital's Rule

$$= \lim_{x \to 0} \frac{-[x\cos x + \sin x \cdot 1] - \sin x + \frac{1}{(1-x)^2}}{2}$$

$$= \lim_{x \to 0} \frac{-x.\cos x - \sin x - \sin x + \frac{1}{(1-x)^2}}{2}$$

$$= \frac{1}{2} \text{ Ans.}$$

(14) Evaluate $\lim_{x \to 0} \frac{x - \sin x}{x^3}$

Ans.→ $\lim_{x \to 0} \frac{x - \sin x}{x^3} \left[\frac{0}{0}\right]$

Hence, from L' Hospital Rule, ~~the~~

$$= \lim_{x \to 0} \frac{1 - \cos x}{3x^2} \left[\frac{0}{0}\right]$$

Hence, from L' Hospital Rule.

$$= \lim_{x \to 0} \frac{-(-\sin x)}{3x^2}$$

$$= \lim_{x \to 0} \frac{\sin x}{6x} \left[\frac{0}{0}\right]$$

by L' Hospital rule

$$= \lim_{x \to 0} \frac{\cos x}{6x}$$

$$= \frac{\cos 0}{6 \times 1} = \frac{1}{6} \text{ Ans.}$$

(15) Evaluate, $\lim_{x \to 0} \frac{x - \tan x}{x^3}$.

Ans.→ $\lim_{x \to 0} \frac{x - \tan x}{x^3}$

$$= \lim_{x \to 0} \frac{x - \frac{\sin x}{\cos x}}{x^3}$$

$$= \lim_{x \to 0} \frac{x \cos x - \sin x}{x^3 \cos x} \left[\frac{0}{0}\right]$$

Hence, from L' Hospital Rule

$$= \lim_{x \to 0} \frac{\cancel{x} \cos x - x \sin x - \cos x}{3x^2 \cos x - x^3 \sin x}$$

$$= \lim_{x \to 0} \frac{-\sin x}{3x \cos x - x^2 \sin x} \left[\frac{0}{0}\right]$$

Hence, from L' Hospital Rule

$$= \lim_{x \to 0} \frac{-\cos x}{3 \times 1 \cos 0 - 3x \sin x - 2x \sin x - x^2 \cos x}$$

$$= \frac{-\cos 0}{3\cos 0 - 0 - 0 - 0}$$

$$= \frac{-1}{3\times 1 - 0 + 0 + 0} = \frac{-1}{3} \text{ Ans.}$$

(16) Evaluate $\lim_{x\to 0} \frac{e^x - e^{\sin x}}{x - \sin x}$

Ans. $\rightarrow \lim_{x\to 0} \frac{e^x - e^{\sin x}}{x - \sin x} \left[\frac{0}{0}\right]$

Hence, from L' Haspital Rule

$$= \lim_{x\to 0} \frac{e^x - e^{\sin x}\cdot \cos x}{1 - \cos x} \left[\frac{0}{0}\right]$$

Hence, from L' Haspital Rule

$$= \lim_{x\to 0} \frac{e^x - e^{\sin x}\cdot \cos x \cdot \cos x + e^{\sin x}\cdot \sin x}{1 - (-\sin x)}$$

$$= \lim_{x\to 0} \frac{e^x - e^{\sin x}\cdot \cos^2 x + e^{\sin x}\cdot \sin x}{\sin x} \left[\frac{0}{0}\right]$$

Hence, from L' Haspital Rule

$$= \lim_{x\to 0} \frac{e^x - e^{\sin x}\cdot \cos^2 x \cdot \cos x + \cancel{e^{\sin x}\cdot \sin x} + e^{\sin x}\cdot \sin^2 x + e^{\sin x}\cdot \sin x \cos x + e^{\sin x}\cdot \cos x}{\cos x}$$

$$= \lim_{x\to 0} \frac{e^x - e^{\sin x}\cdot \cos^3 x + e^{\sin}\cdot \sin^2 x + e^{\sin x}\cdot \sin x \cos x + e^{\sin x}\cdot \cos x}{\cos x}$$

(17) Evaluate $\lim_{x \to 0} \frac{\sin hx - \sin x}{x^3}$

Ans. $\rightarrow \underset{x \to 0}{Lt} \frac{\frac{d}{dx} \sin hx - \sin x}{\frac{d}{dx}(x^3)} \left[\frac{0}{0}\right]$

~~Hence, from L' Hospital Rule,~~

$$\underset{x \to 0}{Lt} \frac{\cos hx - \cos x}{3x^2} \left[\frac{0}{0}\right]$$

Hence, from L' Hospital Rule,

$$\underset{x \to 0}{Lt} \frac{\sin hx + \sin x}{6x} \left[\frac{0}{0}\right]$$

Hence, from L' Hospital Rule,

$$= \underset{x \to 0}{Lt} \frac{\frac{d}{dx}(\sin hx + \sin x)}{\frac{d}{dx}(6x)}$$

$$= \underset{x \to 0}{Lt} \frac{\cos hx + \cos x}{6 \times 1}$$

$$= \underset{x \to 0}{Lt} \frac{\cos h0 + \cos 0}{6} = \frac{1+1}{6} = \frac{\cancel{2}^1}{\cancel{6}_3} = \frac{1}{3} \text{ Ans.}$$

(18) Evaluate $\underset{x \to 0}{Lt} \frac{\sin x + \sin hx - 2x}{x^5}$

Ans. $\rightarrow \underset{x \to 0}{Lt} \frac{\sin x + \sin hx - 2x}{x^5} \left[\frac{0}{0}\right]$

Hence, from L' Hospital Rule

$$\underset{x \to 0}{Lt} \frac{\cos x + \cos x - 2 \times 1}{5x^4} \left[\frac{0}{0}\right]$$

Hence from L' Hospital Rule,

$$= \lim_{x \to 0} \frac{-\sin x + \sinh x}{20x^3} \quad \left[\frac{0}{0}\right]$$

Hence, from L' Hospital Rule,

$$= \lim_{x \to 0} \frac{-\cos x + \cosh x}{60x^2} \quad \left[\frac{0}{0}\right]$$

Hence, from L' Hospital Rule

$$= \lim_{x \to 0} \frac{\sin x + \sinh x}{120x} \quad \left[\frac{0}{0}\right]$$

Hence, from L' Hospital Rule.

$$= \lim_{x \to 0} \frac{\cos x + \cosh x}{120 \times 1} = \frac{1+1}{120} = \frac{\cancel{2}\,1}{\cancel{120}\,60} = \frac{1}{60} \text{ Ans.}$$